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A NEW COMPRESSION METHOD USING A CHAOTIC SYMBOLIC APPROACH

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Abstract - **In this article, a new compression method is investigated using a particular chaotic modulation type. The symbolic approach presented associates to every informational sequence a trajectory in the state space of the chaotic generator. We are also introducing a new type of chaotic generator adapted to the probability distribution of the informational sequence, and we prove that using the described generator the theoretical compression performances attain the optimal entropy compression. Finally we confirm the theoretical analysis with performance tests for different sequences with different symbolic probabilities.**

Index Terms: **compression algorithm, symbolic dynamics, probabilistc Bernoulli generator, binary initial condition**

I. INTRODUCTION

The problem of loss less data compression has received an increased attention both by the soft making companies but also by free-lancer researchers who have developed various types of algorithms, starting from static ones to the more powerful adaptive ones [3].

In opposition with the data compression algorithms, the use of chaotic signals is relatively new in the area of telecommunications and few studies have been made with the application of chaotic signals and generators specifically for data compression and coding. A larger attention has received the cipher technology and just lately by the introduction of the symbolic dynamics approach [1], [2] and the channel coded system considered, have really open the use of Piece Wise Linear Markov maps (PWLM) to the coding methods area.

This paper objective is to provide a method of a loss less compression algorithm and to demonstrate that the proposed algorithm achieves the optimal entropy criteria as level of compression.

The paper is organized as follows. Section II describes briefly the symbolic dynamics method and considers its application to a Bernoulli shift map. In section III the introduction of the probabilistic Bernoulli generator is considered, and finally in section IV the compression algorithm is presented and its optimality is proven.

II. THE SYMBOLIC DYNAMICS

Symbolic dynamics is concerned with the analysis of symbolic sequences that have been generated by a dynamical system. The adaptation of the definitions of symbolic dynamics to chaotic systems allows to partition the infinite number of finite length trajectories into a finite number of trajectory-sets.

This definition given in [1] describes almost completely the general symbolic dynamics approach applied to chaotic sequences. In this section we will not consider all the mathematical apparatus employed in demonstrating the general statistic properties but a particular application to the Bernoulli shift map given by:

$$f(x) = \begin{cases} f^{\{1\}}(x) = 2x, x \in I_{\{1\}} = [0, 0.5] \\ f^{\{2\}}(x) = 2x - 1, x \in I_{\{2\}} = (0.5, 1] \end{cases} \tag{1}$$

In the above equation we have considered the disjoint and connected intervals $I_{\{n\}}$, with $\cap_{n=1}^{N} I_{\{n\}} = \emptyset$, and $\cup_{n=1}^{N} I_{\{n\}} = I$ where in this case $N = 2$, $n = 1...N$ and $I = [0, 1]$. To simply define the symbolic dynamics we consider a sequence $s = \{s_i, i = 1...M | s_i \in S\}$ where S represents the alphabet of symbols available, $S_{\{n\}} \in S$, $n = 1...N$ so to associate to every symbol $S_{\{n\}}$ an interval $I_{\{n\}}$. In this way we have defined a bijective function between the division $\cup_{n=1}^{N} I_{\{n\}} = I$ of the chaotic function state space to the alphabet $\cup_{n=1}^{N} S_{\{n\}} = S$. We have used the terminology of alphabet to make easier the explanation of the compression algorithm in the section IV.

Now for every interval $I_{\{n\}}$ the probability generator associates a linear function $f^{\{n\}}$: $I_{\{n\}} \rightarrow I$ which on the particular interval $I_{\{n\}}$ is bijective and in consequence invertible, so we can affirm that exists $f^{-1\{n\}} : I \rightarrow I_{\{n\}}$ for every $n = 1...N$. As an example, let us consider $s_M = S_{\{2\}}$. We can deterministically identify the interval where the initial condition for the step M has to be. We have no limits for the step $M+1$, so the interval from which are mapping initially is concerning all the possible values (I). The function who can map any of the values of the interval I to the interval $I_{\{2\}}$ is given by $f^{-1\{2\}}$ and it's bijectivity ensures us that for any value in I it will correspond just one value in $I_{\{2\}}$ and vice versa. So finally for the M^{th} symbol we have :

$$I^{\{M\}} = f^{-1\{2\}}(I) = I_{\{2\}} \tag{2}$$

where we have designated with $I^{\{i\}}$ the interval where the initial condition must be so that the symbolic sequence generated to be $\{s_i, s_{i+1}, ..., s_M\}$. Considering now that $s_{M-1} = S_{\{1\}}$ we will have this time as interval of possible values $I^{\{M\}}$, and consequently as interval of possible initial conditions $I^{\{M-1\}} = f^{-1\{1\}}(I^{\{M\}})$. In general we can express the relation (2) for any symbol s_i which can take any value S_n by the following recurrence form:

$$I^{\{i\}} = f^{-1\{n\}}(I^{\{i+1\}}) \tag{3}$$

The application of back propagation relation given in (3) to the Bernoulli map (1) is exemplified in figure 1 for the sequence $S_{\{2\}}, S_{\{1\}}, S_{\{2\}}$.

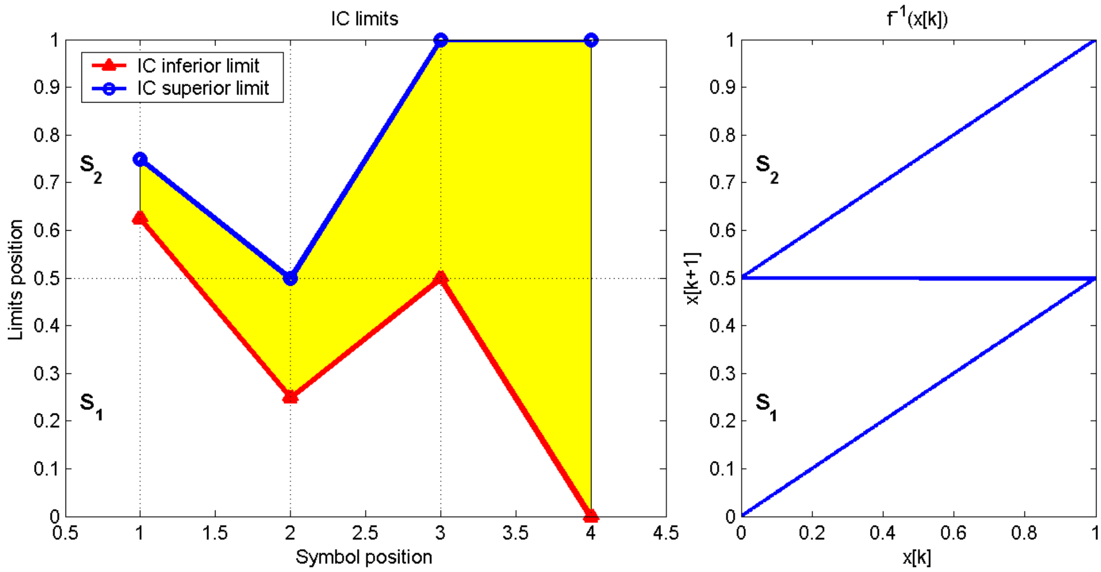

Fig. 1. The interval of possible IC deducted through back propagation

A similar approach has been largely developed by Schweizer and Schimming [1] for a bias-free Bernoulli map, with the direct application to the noise reduction for PWLM generated chaotic sequences.

III. PROBABILISTIC BERNOULLI GENERATOR

In this section we introduce the probabilistic Bernoulli generator as non-uniform piece-wise linear 1^{st} order Markov map. The non-uniformity comes from the idea that the intervals $I_{\{n\}}$ can be unequal.

Now we consider the alphabet of symbols S with the probability of apparition of each symbol given by P_n, $n = 1...N$. We have both for $I_{\{n\}}$ and P_n the relations:

$$\begin{array}{l} \cup_{n=1}^{N} I_{\{n\}} = I = [0,1] \\ \sum_{n=1}^{N} P_n = 1 \\ \cap_{n=1}^{N} I_{\{n\}} = \phi \\ P_n \text{ - probability of disjoint processes (symbols)} \end{array} \tag{4}$$

Using the relations (4) we can very easily develope a relation, between the discrete probability distribution and the intervals that define the state space for the symbolic dynamics, of the following form:

$$\begin{array}{l} I_{\{1\}} = [0, P_1] \\ I_{\{n\}} = (P_{n-1}, P_{n-1} + P_n], n = 2...N \end{array} \tag{5}$$

and also give a characteristic function for the *probabilistic Bernoulli map*:

$$f_p(x) = \begin{cases} f_p^{\{1\}}(x) = \frac{1}{P_1}x, x \in I_{\{1\}} \\ f_p^{\{2\}}(x) = \frac{1}{P_2}\left(x - P_1\right), x \in I_{\{2\}} \\ \ldots.. \\ f_p^{\{N\}}(x) = \frac{1}{P_N}\left(x - \sum_{n=1}^{N-1} P_n\right), x \in I_{\{N\}} \end{cases} \tag{6}$$

A representation of this characteristic function is given in figure 2.

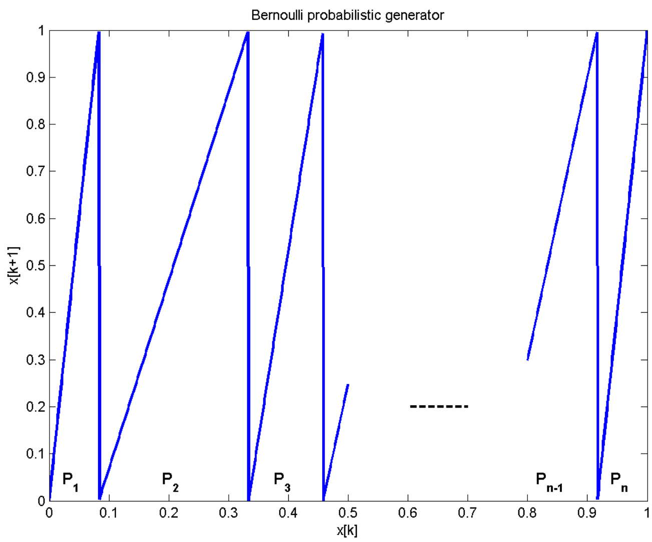

Fig. 2. Bernoulli probabilistic characteristic function

Now the function introduced in (6) has the same properties as the characteristic function of the general Bernoulli map presented in section II, and especially the partial defined function $f_p^{\{n\}} : I_{\{n\}} \to I$ is bijective on the particular interval $I_{\{n\}}$ and in consequence invertible, so we can affirm that exists $f_p^{-1\{n\}} : I \to I_{\{n\}}$ for every $n = 1...N$:

$$\begin{cases} f_p^{-1\{1\}}(x) = P_1 x \\ f_p^{-1\{n\}}(x) = P_n x + \sum_{i=1}^{n-1} P_i \end{cases} \tag{7}$$

The expression given in (7) will be the heart of the compression algorithm presented in section IV, employing the symbolic dynamics method.

IV. CHAOTIC COMPRESSION ALGORITHM

To achieve a certain compression level one has to associate to the compression algorithm some of the statistical properties of the informational signal to be compressed, the goal being to reduce the redundancy and so leaving only the information content. Generally the information of a source code s_i is defined by $-\log_2(p(s_i))$, and consequently there is defined the average information content over the source alphabet as the entropy of that alphabet (8):

$$H = -\sum_{n=1}^{N} P_n \log_2(P_n) \tag{8}$$

The optimality of the code is considered in the sense of the minimum redundancy, which is defined by the difference between the average codeword length and the entropy of the associated alphabet, and so a code is considered asymptotically optimal if for a given probability distribution, the ratio between the average codeword length to the entropy approaches 1 as entropy goes to infinity.

The purpose of introducing the symbolic dynamics in section II was to prove that starting with piece-wise particular generator we can "code", using a bijective recursive family of functions, a sequence of symbols to a deterministically obtained interval of initial conditions. We will prove that using this method and the probabilistic Bernoulli generator introduced in section III we can attain the entropy limit for the code obtained.

The compression is achieved in finding the best initial condition, coded in binary format, that using the particular form of the generator defined in (6) can obtain with the symbolic dynamics approach give the initial sequence. Now if we want to find the performances of this method we have to refer to the minimum necessary binary sequence to code a initial condition in a particular interval. It is known that to represent a value in the interval $I \subset [0, 1)$ of dimension d we need $-\log_2 d$ number of bits, so if we can develop a method for calculating the size of the particular interval that the initial condition has to respect, we can provide a compression performance of the algorithm.

We consider as in section II the sequence $s = \{s_i, i = 1...M | s_i \in S\}$ where S is the alphabet of symbols available, $S_{\{n\}} \in S$, $n = 1...N$, with the symbol apparition probability given by:

$$P_n = \frac{1}{M} card\left\{s_i | s_i = S_{\{n\}}\right\} \tag{9}$$

To this discrete probability distribution, we associate the probabilistic Bernoulli generator given in (6) with the property of partial function invertibility with the form given in eq. (7) and the disjunctivity of the definition intervals $I_{\{n\}}$.

If we consider the back-propagation method we can provide the same recurrent form, for the interval $I^{\{i\}}$ of possible values of initial conditions, as it is given in equation (3); the symbol coded was $s_i = S_{\{n\}}$:

$$I^{\{i\}} = f_p^{-1\{n\}}(I^{\{i+1\}}) \tag{10}$$

Using the relation above and the linear form of $f_p^{-1\{\cdot\}}$ we can provide also a recurrent expression for the size of the interval $I^{\{i\}}$ (we are also working with the hypothesis that the symbol coded was $s_i = S_{\{n\}}$, and $0 \le P_n < 1$):

$$size(I^{\{i\}}) = P_n size(I^{\{i+1\}}) \tag{11}$$

and give the size of possible initial conditions for all the sequence s :

$$size(I^{\{1\}}) = \prod_{i=1}^{M} P(s_i \in S) = \prod_{n=1}^{N} (P_n)^{P_n M} \tag{12}$$

we have used the definition for the discrete probability distribution given in (9). Now we can obtain the number of necessary bits to code the initial condition:

$$-\log_2 \left(size(I^{\{1\}})\right) = -\sum_{n=1}^{N} P_n M \log_2 (P_n) = M \cdot H \tag{13}$$

With the relation (13) the demonstration of the optimality of the compression algorithm is given. A similar classical algorithm was introduced in the 60's by a number of researchers and received the name of arithmetic algorithm. It uses also a $[0,1]$ initial interval but with forward projecting technique and the direct implementations of this algorithm were prohibited by requiring high computational effort, but now there have been proposed some adjustments like a power of 2 alphabet distribution probability representation [3].

The decompression algorithm in our case is very simple, using the determined initial condition together with the probabilistic Bernoulli generator defined by $\{P_n\}$ we can generate a chaotic sequence and just make a state phase separation conformed to the intervals $\{I_{\{n\}}\}$. The problem for this algorithm as for the arithmetic compression method is that it doesn't know when to stop so we have to add to the symbols space, a character of *end-sequence*, which will affect the optimality of compressing process as a whole but by using long sequences the algorithms achieves almost optimality.

To exemplify the functionality of the algorithm we have chosen two sequences with the same length and symbols apparition probability (table 14) to see how the recurrent determination of the interval for the initial condition changes and also the trajectory of the regenerated chaotic sequence (figures 3 a,b).

`Considered sequence #1: ''AABACABCCACBAACAC#'', length:` $M = 18$

Character	Probability	Range $I_{\{n\}}$
A	0.4444	[0, 0.44]
B	0.1667	(0.44, 0.61]
C	0.3333	(0.61, 0.94]
#	0.0556	(0.94, 1]

(14)

`Entropy:` $H = 1.7108$, `normal coding for the IC (initial conditions):` $H \cdot M = 30.7944$

`Coded IC: [0 0 0 1 1 0 0 1 0 0 0 1 0 1 1 0 1 1 0 1 0 1 1 0 0 0 1 1 0 0 1], length 31`

`Uncoded IC:` $x_0 = 0.09800471039489$

`Considered sequence #2: ''ABAACABCACCBACAAC#'', length:` $M = 18$

`Coded IC: [0 0 0 1 1 0 0 1 0 0 0 1 0 1 1 0 1 1 0 1 0 1 1 0 0 0 1 1 0 0 1], length 31`

`Uncoded IC:` $x_0 = 0.20770334312692$

A qualitative explanation of figures 3 a and b, is that using the property of bijectivity of the functions $f_p^{\{n\}}(x)$ ensures us that the regenerated trajectory determined by the initial condition x_0 will be always contained in the back propagation recurrent determination of the $I^{\{i\}}$ intervals, and limited by the inferior and the superior limits defining them. We can observe that for both seqences the trajectory generation stops when detecting *end-sequence* symbol. There can be considered other methods for the decrompression, like a limited length signal encoding permiting only a fix number of symbols to be processed each time, but each method can be particulary good for different informational signals with different discrete probability distributions, a more application directed study needing to be made.

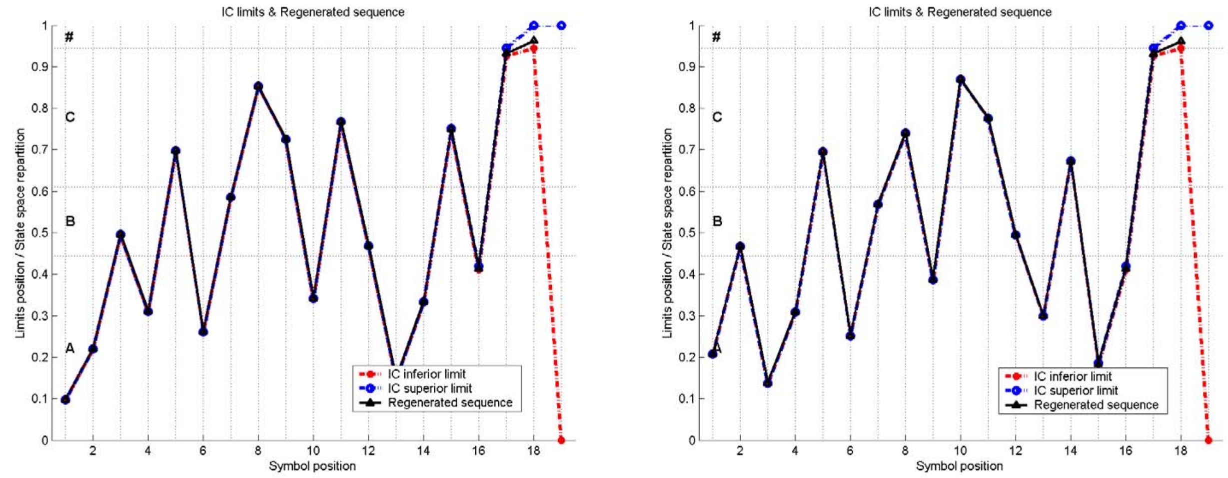

Fig. 3. Trajectory limits to determine the recurrent interval of IC & regenerated trajectory for: a) sequence #1, b) sequence #2

V. CONCLUSIONS

An optimal entropy criteria loss-less compression algorithm was introduced as a possibly new application of chaotic signals to coding procedure. The use of chaotic dynamics and a special type of a chaotic generator, called probabilistic Bernoulli generator were considered to map a sequence into an initial condition associated with the generator. Only the static compression algorithm is considered in this article, an adaptive form and/or enriched Markov chain order can be envisaged to improve performances. The use of the PWLM map permits a fast reconstruction of the initial sequence making it very interesting for real time powerful implementations.

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[3] U.S. Patent 6,560,368 *Arithmetic Coding and Decoding Methods and Related Systems*