Kinematics

Master SIIA
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Introduction

What is kinematics?

Kinematics: the science of motion, that treats motion of bodies without regard to the forces which cause the motion.

Robot kinematics describes the **pose and its derivatives** (velocity, acceleration, jerk, snap, etc.) for the bodies that comprise a mechanism.

Foundational element for Dynamics, Motion Planning, and Motion Control algorithms.

Course objectives

- Understand basic **kinematics for robotics**
- Discover **tools for kinematic computations**
- Find **Bibliographical references** for further reading
- Practice session

References

Course based on the following sources:

- **Modern Robotics** (K. Lynch, F. Park, 2017)
- **Introduction to robotics: mechanics and control** (John J. Craig, 2017)
- **Handbook of robotics** (editors B. Siciliano, O. Khatib, 2nd ed. 2017)
- **Robotics manipulation (course)** (Russ Tedrake)

Questions answered by the course

- How to describe positions, orientations, and frames?
- How to apply matrix transformations?
- How to describe manipulator link connections?

Course structure

1. Notation
2. Spatial descriptions
3. Geometric transformations (rigid body motions)
    1. Translation, Rotation, Transformation
4. 3D kinematic chains
    1. Kinematic joints
    2. Denavit-Hartenberg notation
    3. Universal Robot Description Format (URDF)
    4. Forward kinematics
    5. Inverse kinematics

Notation

(source: **Robotic Manipulation** by Russ Tedrake)

Notation

1. Points

Description	Notation
examples of points:	A, C, B_{cm}
position vector of point A:	p^A
position of point C measured from A:	${}^Ap^C$
position of point C measured from A, expressed in frame F:	${}^Ap^C_F$
x component of the position of point C measured from A, expressed in frame F:	${}^Ap^C_{F_x}$

Notation

2. Frames

Description	Notation
world frame:	W
body frame of Body i:	B_i
position of point A measured from the origin of the world frame, expressed in the world frame:	${}^{W}p_{W}^{A}$
position of point A measured from the origin of frame F, expressed in the frame F:	${}^{F}p_{F}^{A} \equiv {}^{F}p^{A}$
if the "measured from" field is omitted, then we assume that the point is measured from W:	$p^{A} \equiv {}^{W}p_{W}^{A}$

Notation

3. Points and frames: summary

$$\textit{measured from}\rightarrow {}^{B}p^{A\leftarrow\textit{target (point or frame)}}_{C\leftarrow\textit{for vectors: expressed in (frame)}}$$

$$\textit{quantity type}(p,R,X,\ldots)\nearrow$$

Notation

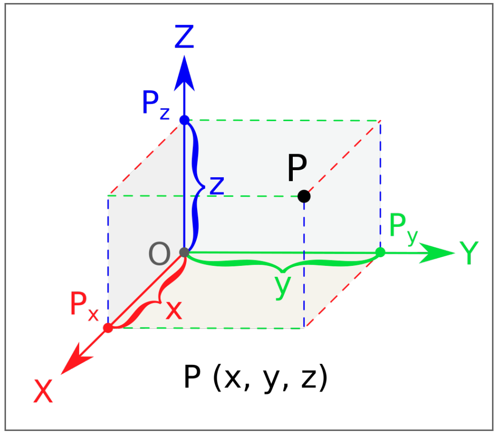
4 . Frames for rotations

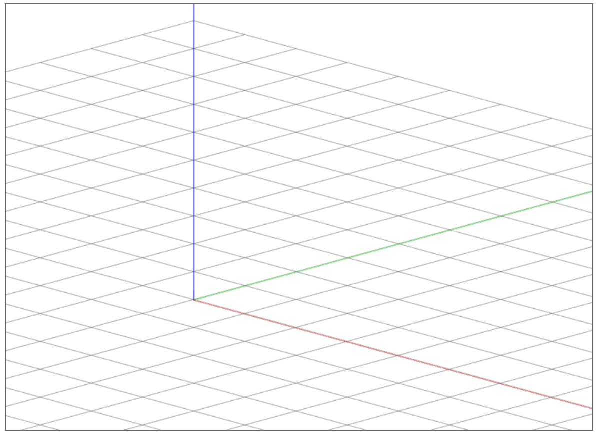
Description	Notation
rotation of frame A measured from frame B:	${}^{B}R^{A}$
pose of frame A measured from frame B:	${}^{B}X^{A}$
pose of an object O:	${}^{B}X^{A}$

Spatial descriptions

COORDINATE FRAMES

Coordinate frame: consists of an origin O_i and a triad of mutually orthogonal basis vectors $(\hat{x}_i, \hat{y}_i, \hat{z}_i)$ that are all fixed within a body.



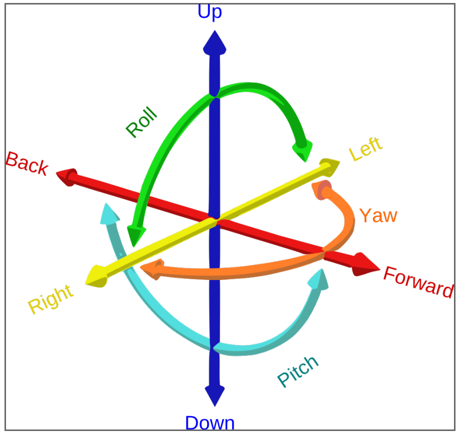


Spatial descriptions

POSE: POSITION AND ORIENTATION

Description	Notation
position of A with respect to frame W: (equivalent to a translation of p^W by ${}^W p^A$)	${}^W p^A = \left({}^W x^A, {}^W y^A, {}^W z^A\right)^T$
orientation of A with respect to frame W: (equivalent to a rotation of R^W by ${}^W R^A$)	${}^W R^A = \left({}^W \hat{x}^A, {}^W \hat{y}^A, {}^W \hat{z}^A\right)$

pose = (position, orientation)

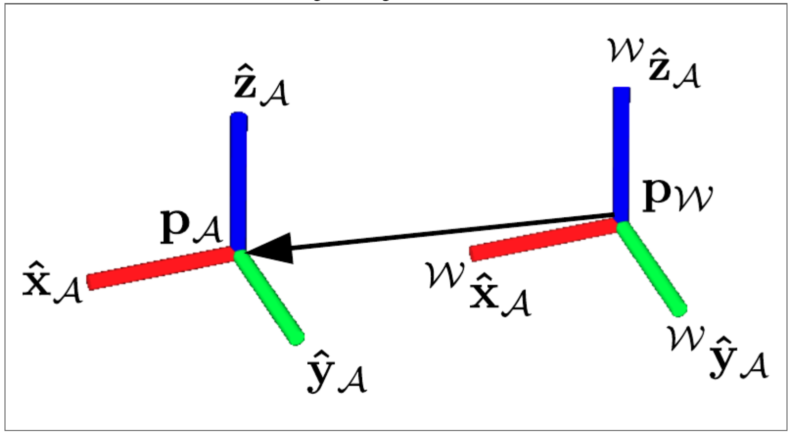


Geometric transformations (rigid-body motions)

TRANSLATION

Translation: displacement which moves every point of a body by the same amount in a given direction.

- Translation of 3D position $\mathbf{p} = (x, y, z)$ by some $\mathbf{t} = (x_t, y_t, z_t) \in \mathbb{R}^3$:
  $\mathbf{p} + \mathbf{t} = (x, y, z) + (x_t, y_y, z_y) = (x + x_t, y + y_t, z + z_t)$



Geometric transformations (rigid-body motions)

3D ROTATION: REPRESENTATIONS

- 3x3 Rotation matrices
- Euler angles (in a rotating or fixed frame)
- Axis angle (a.k.a. "exponential coordinates")
- Unit **quaternions** (solve the problem of singularities)

Geometric transformations (rigid-body motions)

3D ROTATION: CONVERSIONS BETWEEN REPRESENTATIONS [*]

Rotation matrix:

$$ {}^j\mathbf{R}_i = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} $$

Z-*Y*-*X* Euler angles $(\alpha, \beta, \gamma)^T$:

$$ \beta = \mathrm{Atan2}\left(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2}\right) $$

$$ \alpha = \mathrm{Atan2}\left(\frac{r_{21}}{\cos\beta}, \frac{r_{11}}{\cos\beta}\right) $$

$$ \gamma = \mathrm{Atan2}\left(\frac{r_{32}}{\cos\beta}, \frac{r_{33}}{\cos\beta}\right) $$

X-*Y*-*Z* fixed angles $(\psi, \theta, \phi)^T$:

$$ \theta = \mathrm{Atan2}\left(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2}\right) $$

$$ \psi = \mathrm{Atan2}\left(\frac{r_{21}}{\cos\theta}, \frac{r_{11}}{\cos\theta}\right) $$

$$ \phi = \mathrm{Atan2}\left(\frac{r_{32}}{\cos\theta}, \frac{r_{33}}{\cos\theta}\right) $$

Angle axis $\theta\hat{\boldsymbol{w}}$:

$$ \theta = \cos^{-1}\left(\frac{r_{11}+r_{22}+r_{33}-1}{2}\right) $$

$$ \hat{\boldsymbol{w}} = \frac{1}{2\sin\theta}\begin{pmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{pmatrix} $$

Unit quaternions $(\epsilon_0\ \epsilon_1\ \epsilon_2\ \epsilon_3)^T$:

$$ \epsilon_0 = \frac{1}{2}\sqrt{1 + r_{11} + r_{22} + r_{33}} $$

$$ \epsilon_1 = \frac{r_{32}-r_{23}}{4\epsilon_0} $$

$$ \epsilon_2 = \frac{r_{13}-r_{31}}{4\epsilon_0} $$

$$ \epsilon_3 = \frac{r_{21}-r_{12}}{4\epsilon_0} $$

Geometric transformations (rigid-body motions)

3X3 ROTATION MATRICES

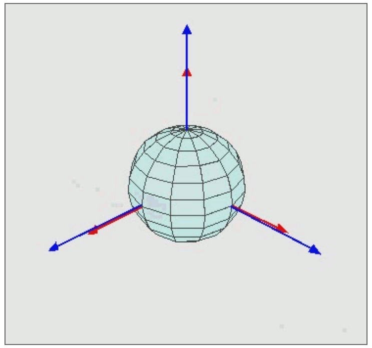
$$
{}^G R^F = \begin{pmatrix} {}^G x^F & {}^G y^F & {}^G z^F \\ x_x & y_x & z_x \\ x_y & y_y & z_y \\ x_z & y_z & z_z \end{pmatrix}
$$

The group of rotation matrices, also known as the **s**pecial **o**rthogonal group **SO(3)**, is the set of all 3 × 3 real matrices R that satisfy:

- $R^T R = I$, and
- $det(R) = 1$

Geometric transformations (rigid-body motions)

EULER ANGLES: ROTATING FRAME (1)



*z-x-z rotation (source: **Wikipedia**)*

Orientation is given by 3 successive rotations (α, β, γ) around a **predefined** order of coordinate axes (e.g. Z-Y-X) of a **rotating** frame

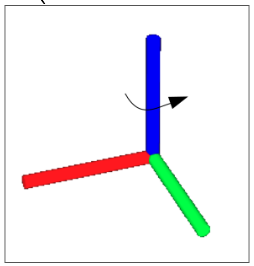
Geometric transformations (rigid-body motions)

EULER ANGLES: ROTATING FRAME (2)

$$R = R_z(\alpha)R_y(\beta)R_x(\gamma)$$

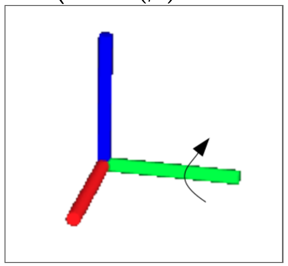
Z-axis rotation

$$R_z(\alpha) = \begin{pmatrix} cos(\alpha) & -sin(\alpha) & 0 \\ sin(\alpha) & cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



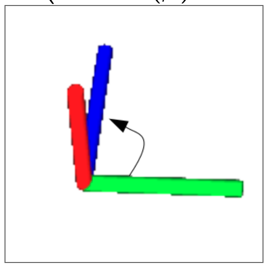
Y-axis rotation

$$R_y(\beta) = \begin{pmatrix} cos(\beta) & 0 & sin(\beta) \\ 0 & 1 & 0 \\ -sin(\beta) & 0 & cos(\beta) \end{pmatrix}$$



X-axis rotation

$$R_x(\gamma) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos(\beta) & -sin(\beta) \\ 0 & sin(\beta) & cos(\beta) \end{pmatrix}$$

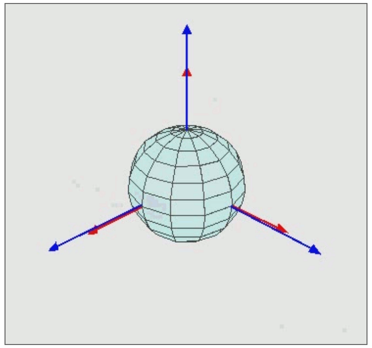


Attention! SO(3) rotations are NOT commutative!

For $R_1, R_2 \in SO(3), R_1R_2 \neq R_2R_1$

Geometric transformations (rigid-body motions)

EULER ANGLES: ROTATING FRAME (3)



*z-x-z rotation (source: **Wikipedia**)*

12 possible rotation orderings:

- Proper Euler angles: x-y-x, x-z-x, y-x-y, y-z-y, z-x-z, z-y-z
- Tait-Bryan angles: x-y-z, x-z-y, y-x-z, y-z-x, z-x-y, z-y-x

Geometric transformations (rigid-body motions)

EULER ANGLES: ROTATING FRAME (4) SINGULARITIES

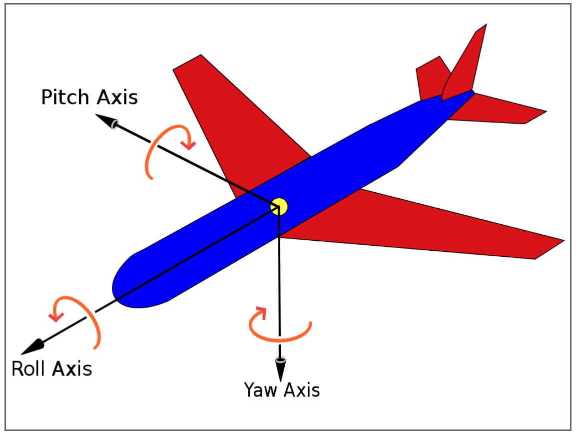
- Singularities occur when the first and last rotations both occur about the same axis.
- For Tait-Bryan angle orderings (e.g. X-Y-Z, Z-Y-X), angles α and γ are undefined when $\beta = \pm 90^\circ$.
- For proper Euler angle orgerings (e.g. Z-Y-Z, Z-X-Z) the singularity occurs when the second rotation is 0° or 180°.
- Problem relating angular velocity vector of a body to the time derivatives of Euler angles. This velocity relationship for Z-Y-X Euler angles is:

$$\begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{pmatrix} = \frac{1}{cos(\beta)} \begin{pmatrix} 0 & sin(\gamma) & cos(\gamma) \\ 0 & cos(\gamma)cos(\beta) & -sin(\gamma)cos(\beta) \\ cos(\beta) & sin(\gamma)sin(\beta) & cos(\gamma)sin(\beta) \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$ where $(\omega_x, \omega_x, \omega_z)^T = {}^i\omega_i$ is given in moving frame i.

Geometric transformations (rigid-body motions)

EULER ANGLES: FIXED FRAME (1)

- Orientation is given by 3 successive rotations (α, β, γ) around a **predefined** order of coordinate axes (e.g. X-Y-Z) of a **fixed** frame
- Following the **X-Y-Z order convention**, the (α, β, γ) angles are called:
  - **Roll**: a counter-clockwise rotation around the X-axis
  - **Pitch**: a counter-clockwise rotation around the Y-axis
  - **Yaw**: a counter-clockwise rotation around the Z-axis



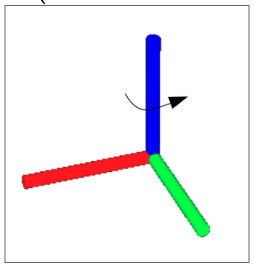
Geometric transformations (rigid-body motions)

EULER ANGLES: FIXED FRAME (2) REMINDER ON ROTATIONS

$$R = R_z(\alpha)R_y(\beta)R_x(\gamma)$$

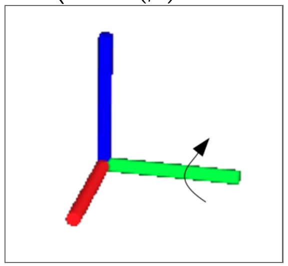
Z-axis rotation

$$R_z(\alpha) = \begin{pmatrix} cos(\alpha) & -sin(\alpha) & 0 \\ sin(\alpha) & cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



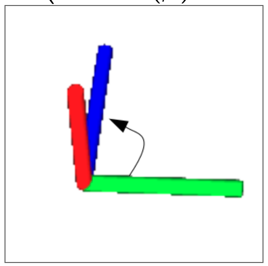
Y-axis rotation

$$R_y(\beta) = \begin{pmatrix} cos(\beta) & 0 & sin(\beta) \\ 0 & 1 & 0 \\ -sin(\beta) & 0 & cos(\beta) \end{pmatrix}$$



X-axis rotation

$$R_x(\gamma) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos(\beta) & -sin(\beta) \\ 0 & sin(\beta) & cos(\beta) \end{pmatrix}$$



Geometric transformations (rigid-body motions)

EULER ANGLES: FIXED FRAME (2)

$$R(\alpha,\beta,\gamma) = R_z(\alpha)\, R_y(\beta)\, R_x(\gamma) =$$
$$\begin{pmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{pmatrix}$$

Rotation performing first $R_x(\gamma)$, then $R_y(\beta)$, then $R_z(\alpha)$

- A fixed frame rotation (α, β, γ) defines a rotating frame rotation (γ, β, α) of opposite axis ordering, and vice versa.
  For example (α, β, γ) with X-Y-Z fixed frame rotation is equivalent to (γ, β, α) with Z-Y-X rotating frame rotation.

Geometric transformations (rigid-body motions)

EULER ANGLES: FIXED FRAME (3) GYROSCOPE

Geometric transformations (rigid-body motions)

Euler angles: Fixed frame (4) Gyroscope

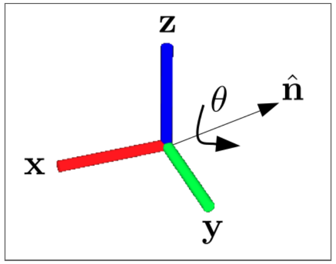
Geometric transformations (rigid-body motions)

EULER ANGLES: FIXED FRAME (5) GIMBAL LOCK

Geometric transformations (rigid-body motions)

AXIS-ANGLE REPRESENTATION (1)

Composed of a rotation angle θ and a 3D unit vector $\hat{n}$ around which the rotation is performed.

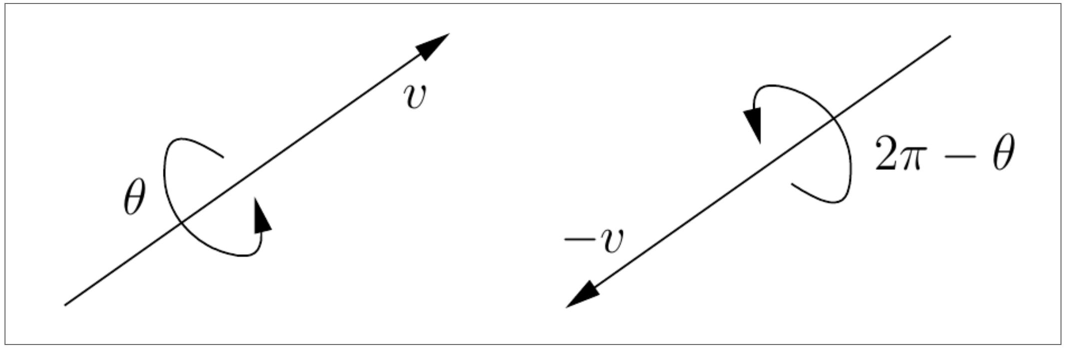


Euler's theorem (rotations): Any displacement of a rigid body such that a point on the rigid body, say O, remains fixed, is equivalent to a rotation about a fixed axis through the point O. [**proof**]

Geometric transformations (rigid-body motions)

AXIS-ANGLE REPRESENTATION (2)

Composed of a rotation angle θ and a 3D unit vector $\hat{n}$ around which the rotation is performed.



There are two ways to encode the same rotation.

Geometric transformations (rigid-body motions)

QUATERNION REPRESENTATION (1)

A quaternion $\mathbf{Q}$ is a $\mathbb{R}^4$ vector represented by:

- a **scalar** part q_0 and
- a **vector** part $\mathbf{q} = (q_1, q_2, q_3)$ where $q_0, q_1, q_2, q_3 \in \mathbb{R}$,

defined as :

$$\mathbf{Q} = q_0 + q_1 i + q_2 j + q_3 k$$

Geometric transformations (rigid-body motions)

Quaternion representation (2)

Rules

$ii = kk = kk = ijk = -1$

$ij = k, jk = i, ki = j$

$ji = -k, kj = -i, ik = -j$

Multiplication: associative, NON commutative

$\mathbf{PQ} = (p_0 + p_1 i + p_2 j + p_3 k)(q_0 + q_1 i + q_2 j + q_3 k)$

Conjugation:

$\mathbf{Q}^* = q_0 - q_1 i - q_2 j - q_3 k$

$\mathbf{QQ}^* = q_0^2 + q_1^2 + q_2^2 + q_3^2$

Norm:

$||\mathbf{Q}|| = \sqrt{\mathbf{QQ}^*}$

Unit quaternion:

$\mathbf{Q}' = \frac{\mathbf{Q}}{||\mathbf{Q}||}$

$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1$

Identity quaternion:

$\mathbf{I} = 1 + 0i + 0j + 0k$

$\mathbf{QI} = \mathbf{Q}$

Geometric transformations (rigid-body motions)

QUATERNION REPRESENTATION (3)

How to convert from Axis-angle representation to Quaternion representation?

The rotation of a point $\mathbf{p}$ around an axis defined by the vector $\mathbf{v} = (v_1, v_2, v_3)$ for θ degrees is represented by a quaternion with the following values:

$q_0 = cos(\theta/2)$
$q_1 = v_1 sin(\theta/2)$
$q_2 = v_2 sin(\theta/2)$
$q_3 = v_3 sin(\theta/2)$
$\mathbf{Q} = q_0 + q_1 i + q_2 j + q_3 k$

Geometric transformations (rigid-body motions)

QUATERNION REPRESENTATION (4)

How to convert a unit quaternion $\mathbf{Q} = q_0 + q_1 i + q_2 j + q_3 k$ to an SO(3) rotation?

$$R(\mathbf{Q}) = \begin{pmatrix} 2(q_0^2 + q_1^2) - 1 & 2(q_1 q_2 - q_0 q_3) & 2(q_1 q_3 + q_0 q_2) \\ 2(q_1 q_2 + q_0 q_3) & 2(q_0^2 + q_2^2) - 1 & 2(q_2 q_3 - q_0 q_1) \\ 2(q_1 q_3 - q_0 q_2) & 2(q_2 q_3 + q_0 q_1) & 2(q_0^2 + q_3^2) - 1 \end{pmatrix}$$

Geometric transformations (rigid-body motions)

QUATERNION REPRESENTATION (5)

How to apply a quaternion rotation $\mathbf{Q}$ to a point $\mathbf{p}_1 = (x_1, y_1, z_1)$?

1. Write point $\mathbf{p}_1$ as a quaternion: $(0, x_1 i, y_1 j, z_1 k)$
2. Product (rotation result) is given by: $\mathbf{QpQ}*$
   and is of the form $(0, x_2, y_2, z_2)$, where $\mathbf{p}_2 = (x_2, y_2, z_2)$ is the location where point $\mathbf{p}_1$ ends up after rotation.

Geometric transformations (rigid-body motions)

Spatial Algebra

Description	Notation
Positions expressed in the same frame can be added when their reference and target symbols match	${}^{A}p_{F}^{B} + {}^{B}p_{F}^{C} = {}^{A}p_{F}^{C}$
Addition is commutative, and the additive inverse is well defined	${}^{A}p_{F}^{B} = -{}^{B}p_{F}^{A}$
Multiplication by a rotation can be used to change the "expressed in" frame:	${}^{A}p_{G}^{B} = {}^{G}R^{F}\,{}^{A}p_{F}^{B}$
Rotations can be multiplied when their reference and target symbols match:	${}^{A}R^{B}\,{}^{B}R^{C} = {}^{A}R^{C}$
Inverse operation for rotation:	$[{}^{A}R^{B}]^{-1} = {}^{B}R^{A}$
Inverse operation for rotation:	$R^{-1} = R^{T}$

Transformations:	${}^{G}p^{A} = {}^{G}T^{F}\,{}^{F}p^{A} = {}^{G}p^{F} + {}^{F}p^{A}_{G} = {}^{G}p^{F} + {}^{G}R^{F}\,{}^{F}p^{A}$
Transformation composition:	${}^{A}T^{B}\,{}^{B}T^{C} = {}^{A}T^{C}$
Transformation inverse:	$\left[{}^{A}T^{B}\right]^{-1} = {}^{B}T^{A}$

Geometric transformations (recap 1)

Position: 3D vector ${}^W\mathbf{p}^A = (x, y, z) \in \mathbb{R}^3$ for positioning a frame
Orientation: 3×3 matrix ${}^W R^A$ for orienting a frame
Translation: associative, commutative

- Translation of 3D position $\mathbf{p} = (x, y, z)$ by some $\mathbf{t} = (x_t, y_t, z_t) \in \mathbb{R}^3$: $\mathbf{p} + \mathbf{t} = (x, y, z) + (x_t, y_y, z_y) = (x + x_t, y + y_t, z + z_t)$
- Translations are NOT commutative with Rotations

Geometric transformations (recap 2)

Rotation: $\mathbf{Rp}$

- Mobile frame: $R = R_z(\alpha)R_y(\beta)R_x(\gamma) = R(\alpha, \beta, \gamma)$
- Fixed frame: $R = R_x(\gamma)R_y(\beta)R_z(\alpha) = R(\gamma, \beta, \alpha)$
- Quaternion numbers: $R(Q)p = QpQ*$
- Rotations are NOT commutative

Geometric transformations (recap 3)

Tranformation (Roto-translation):

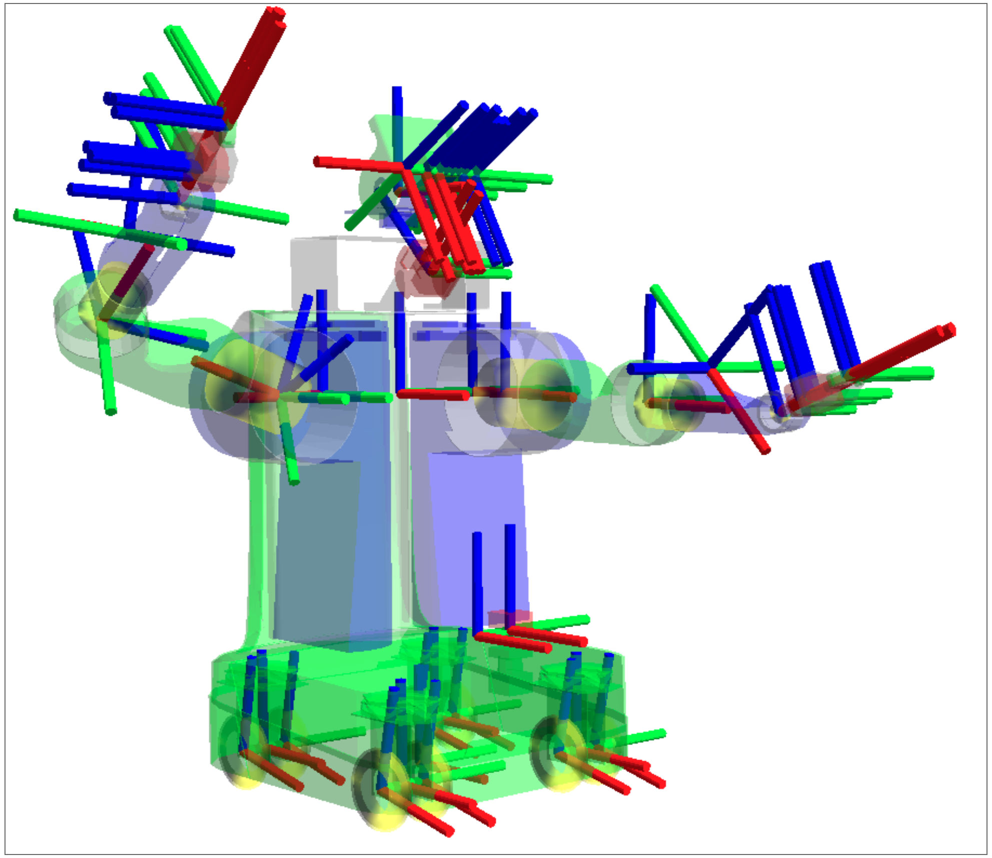
- Homogeneous transformation matrix for 3D bodies: 4x4 matrix that performs the rotation given by $\mathbf{R}(\alpha, \beta, \gamma)$ followed by a translation given by $\mathbf{t} = (x_t, y_t, z_t)$
- Homogeneous transformation matrix: ${}^{W}\mathbf{T}^{A} = \begin{pmatrix} \mathbf{R} & \mathbf{t} \\ 0^T & 1 \end{pmatrix}$

  where $\mathbf{R} \in \mathbf{SO(3)}$ and $\mathbf{t} \in \mathbb{R}^3$

$$T = \begin{pmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma & x_t \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma & y_t \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma & z_t \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Usage: $\mathbf{T} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$

3D kinematic chains



3D kinematic chains

DEFINITION

A robot mechanism is a system of (rigid) bodies connected by joints.

A 3D kinematic chain:

- is a sequence composed of rigid bodies $A_i, i = 1, 2, \ldots, m$ called links
- each link $A_i, i = 1, 2, \ldots, m-1$ is attached to link A_{i+1} allowing a constrained motion of A_{i+1} with respect to A_i
- The place where A_i is attached to A_{i+1} is called a joint

A joint:

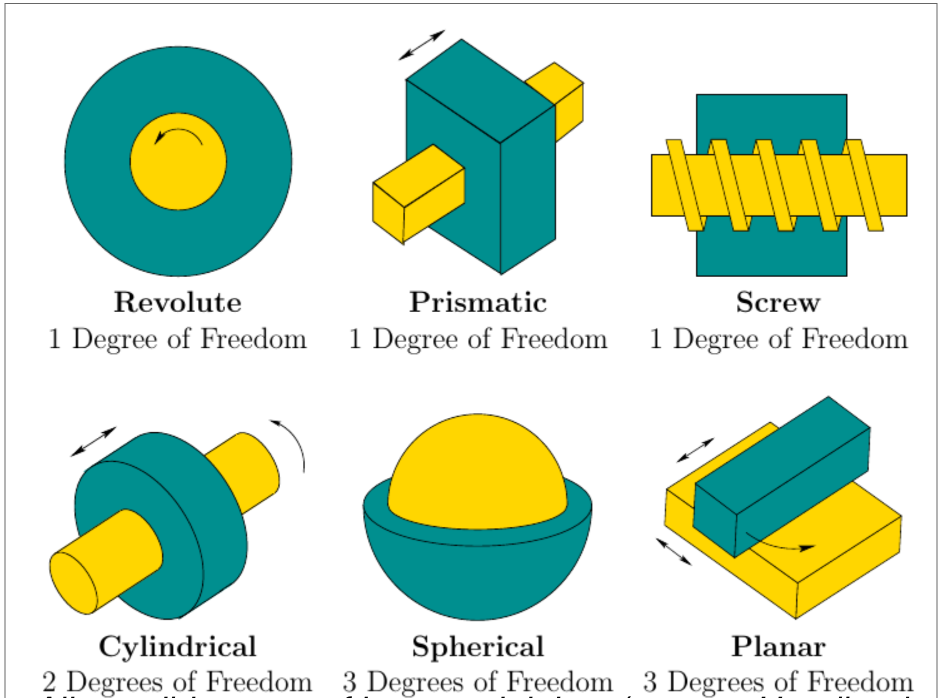
- specifies the constrained motion of a frame fixed in one link of the joint, relative to a frame fixed to the other link.
- is defined by the rotation matrix, position vector, free modes, constrained modes

3D kinematic chains

KINEMATIC JOINTS

Joint types:

- Revolute (hinge), 1 DOF
- Prismatic (sliding), 1 DOF
- Screw (helical), 1 DOF
- Cylindrical, 2 DOF
- Spherical, 3 DOF
- Planar, 3 DOF



All possible types of lower-pait joints (source: Handbook of Robotics)

3D kinematic chains

DENAVIT-HARTENBERG NOTATION (1)

Idea:

- attach reference frames to each link of the open chain,
- derive the forward kinematics from the knowledge of the relative displacements between adjacent link frames.

Links (and their reference frames) numbered sequentially from 0 to n:

- ground link is labeled 0,
- end-effector frame is attached to link n.

The forward kinematics of the n-link open chain expressed as:
$T_{0,n}(\theta_1, \ldots, \theta_n) = T_{0,1}(\theta_1)T_{1,2}(\theta_2)\cdots T_{n-1,n}(\theta_n)$
where $T_{i-1,i}$ are homogeneous transforms that denote the relative displacement between link frames $(i-1)$ and (i).

3D kinematic chains

DENAVIT-HARTENBERG NOTATION (2)

Assigning link frames:

1. Determine $\hat{z}_i$-axis:
   for joint i, the joint axis defines the $\hat{z}$-axis of the i-th link frame.
2. Determine the origin of each (i-1) link reference frame:
   a point on a line perpendicular to both joint axes (i-1) and (i), at the place where it intersects joint axis (i-1).
3. Determine the remaining $\hat{x}$- and $\hat{y}$-axes of each link:
   - $\hat{x}$-axis set in the direction of the mutually perpendicular line pointing from (i-1) axis to the next (i) axis.
   - $\hat{y}$-axis determined from the cross product $\hat{x} \times \hat{y} = \hat{z}$ using the right hand rule.

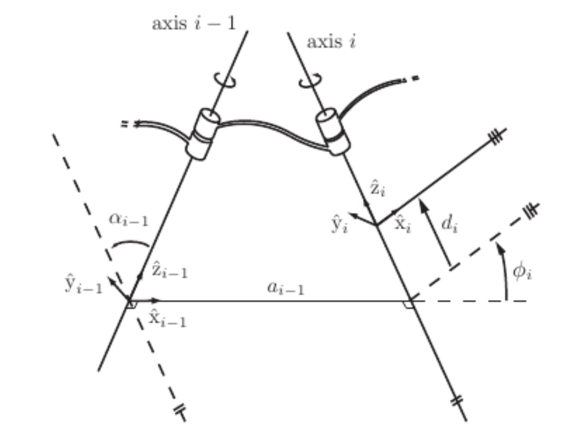


Figure C.1: Illustration of the Denavit–Hartenberg parameters.
*(source: **Modern Robotics** (K. Lynch, F. Park, 2017))*

3D kinematic chains

DENAVIT-HARTENBERG NOTATION (3)

Parameters that specify exactly $T_{i-1,1}$:

- link length a_{i-1}: distance from $\hat{z}_{i-1}$ to $\hat{z}_i$ along $\hat{x}_{i-1}$
- link twist α_{i-1}: angle from $\hat{z}_{i-1}$ to $\hat{z}_i$, measured about $\hat{x}_{i-1}$
- link offset d_i: distance from $\hat{x}_{i-1}$ to $\hat{x}_i$ along $\hat{z}_i$
- joint angle θ_i: the angle from $\hat{x}_{i-1}$ to $\hat{x}_i$, measured about the $\hat{z}_i$-axis.

For special cases, see **Modern Robotics** (K. Lynch, F. Park, 2017).

3D kinematic chains

Representation in ROS: URDF files

Universal Robot Description Format (URDF): an XML (eXtensible Markup Language) file format used by the Robot Operating System (ROS) to describe the kinematics, inertial properties, and link geometry of robots.

A URDF file describes:

- joints (origin frame, parent link and child link, joint type)
- links (origin frame, geometrical shape, mass, center of mass, inertia matrix)

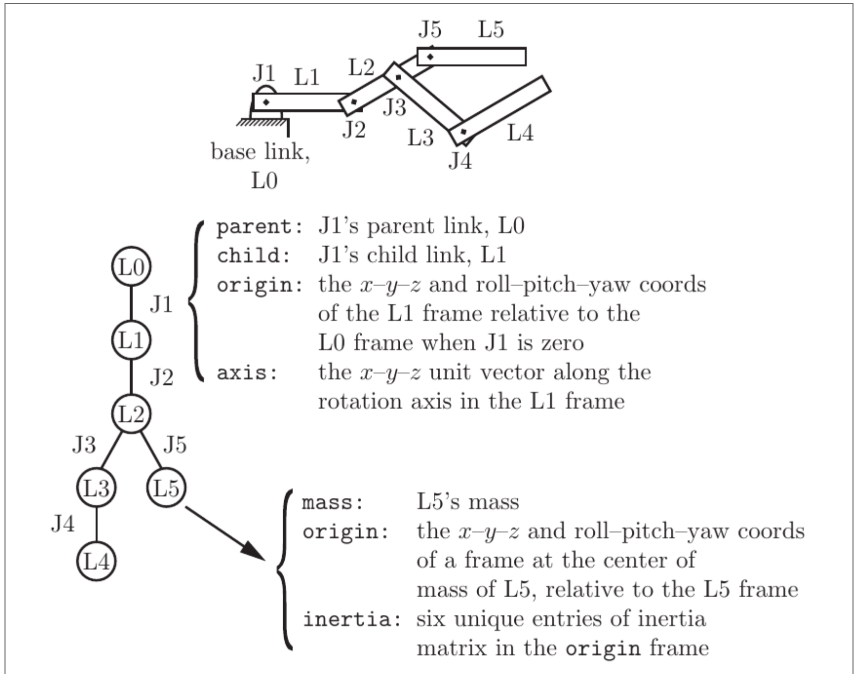


Figure 4.10: A five-link robot represented as a tree, where the nodes of the tree are the links and the edges of the tree are the joints.

Forward kinematics

Forward kinematics of a robot: calculation of the position and orientation of its end-effector frame relative to the base, given the positions of all of the joints and the values of all of the geometric link parameters.

- Can be written as the product of homogeneous transformation matrices of links leading from the base to the end-effector:

$\mathbf{T}_{0,\mathrm{n}} = \mathbf{T}_0 \dots \mathbf{T}_n$

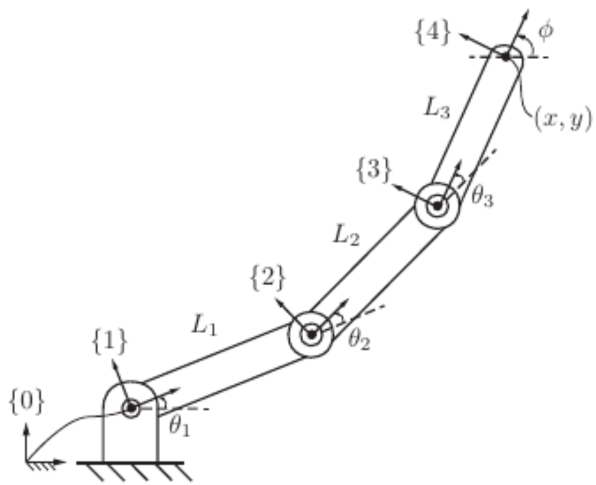
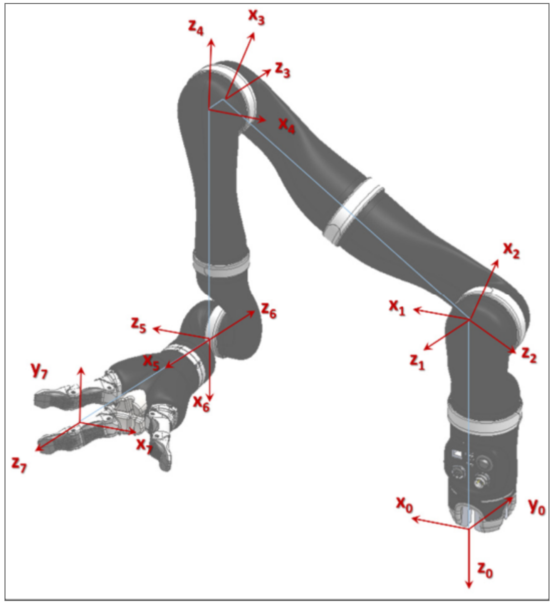


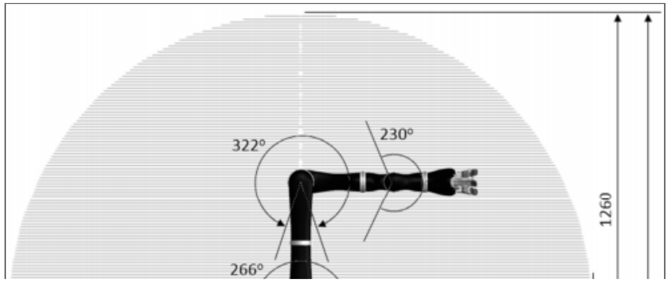
Figure 4.1: Forward kinematics of a 3R planar open chain. For each frame, the x̂- and ŷ-axis is shown; the ẑ-axes are parallel and out of the page.

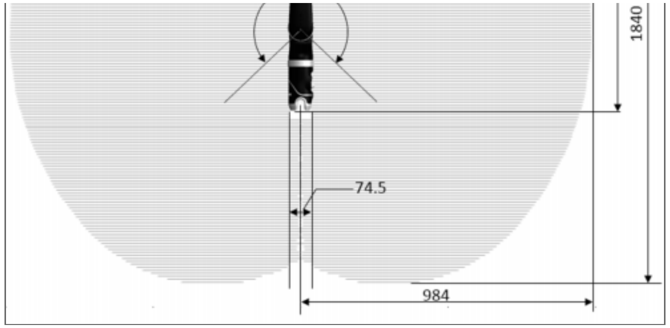
Forward kinematics

Workspace

Workspace: the total volume swept by the robotic mechanism upon execution of all possible motions.
Reachable workspace: total locus of points at which the end-effector can be placed.
Dexterous workspace: points at which the end-effector can be placed while having an arbitrary orientation.






1840
74.5
984

Inverse kinematics (1)

Inverse kinematics given the relative positions and orientations of two members of a mechanism, find the values of all of the joint positions.

- requires solving sets of nonlinear equations
- no solution may exist
- multiple solutions may exist
  (due to robot articulation redundancy; causes singularities)
- the desired position and orientation of the end-effector must lie in the workspace of the manipulator
- numerical methods are required

Inverse kinematics (2)

The design of most robotic articulating mechanism is such that it allows closed-form inverse kinematics solutions.
Sufficient conditions for a 6DOF manipulator to have closed-form inverse kinematics solutions:

- Three consecutive revolute joint axes intersect at a common point, as in a spherical wrist.
- Three consecutive revolute joint axes are parallel.

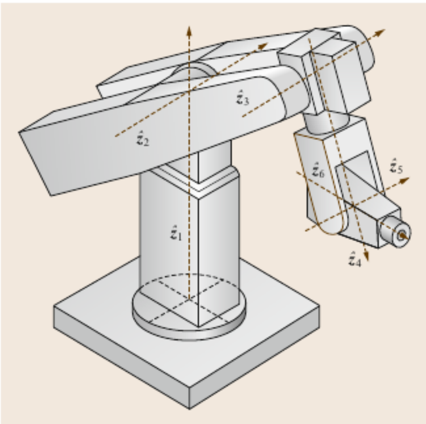


Fig. 2.3 Example six-degree-of-freedom serial chain manipulator composed of an articulated arm with no joint offsets and a spherical wrist

(source: Handbook of robotics)

Questions ?

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